The Actuarial Profession
making financial sense of the future

British Mathematical Olympiad

Round 1 : Wednesday, 5 December 2001

Time allowed *Three and a half hours.*

Instructions

- *Full written solutions - not just answers - are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Do not hand in rough work.*
- *One* complete *solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all five problems.*
- *Each question carries 10 marks.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top* left *hand corner and your name, initials and school in the top* right *hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by the questions 1,2,3,4,5 in order.*
- *Staple all the pages neatly together in the top* left *hand corner.*

Do not turn over until **told to do so.**

The Actuarial Profession
making financial sense of the future

2001 British Mathematical Olympiad
Round 1

1. Find all positive integers m, n, where n is odd, that satisfy
$$\frac{1}{m} + \frac{4}{n} = \frac{1}{12}.$$

2. The quadrilateral $ABCD$ is inscribed in a circle. The diagonals AC, BD meet at Q. The sides DA, extended beyond A, and CB, extended beyond B, meet at P.
Given that $CD = CP = DQ$, prove that $\angle CAD = 60°$.

3. Find all positive real solutions to the equation
$$x + \left\lfloor \frac{x}{6} \right\rfloor = \left\lfloor \frac{x}{2} \right\rfloor + \left\lfloor \frac{2x}{3} \right\rfloor,$$
where $\lfloor t \rfloor$ denotes the largest integer less than or equal to the real number t.

4. Twelve people are seated around a circular table. In how many ways can six pairs of people engage in handshakes so that no arms cross?
(Nobody is allowed to shake hands with more than one person at once.)

5. f is a function from $\mathbb{Z}^+$ to $\mathbb{Z}^+$, where $\mathbb{Z}^+$ is the set of non-negative integers, which has the following properties:-
a) $f(n+1) > f(n)$ for each $n \in \mathbb{Z}^+$,
b) $f(n + f(m)) = f(n) + m + 1$ for all $m, n \in \mathbb{Z}^+$.
Find all possible values of $f(2001)$.